Diagnosis of contextual awareness in mathematics to support the teaching of mathematical modeling

Marta Mikite, Ilze France, Ģirts Burgmanis
University of Latvia
Careful diagnosis of students' performance enables planning support for each student when teaching mathematical modeling. Contextual awareness is essential in every step of mathematical modeling and its absence can lead to blockages in the modeling process. The authors propose an assessment tool that allows the diagnose of contextual awareness. Comparing the number of students who perform the required calculations and the number of students who demonstrate contextual awareness with overall performance in mathematics, the authors find that there is a large proportion of high-performing students who do not demonstrate contextual understanding. There are also a large number of students who demonstrate contextual understanding but do not make the necessary calculations. This variation in students' performance points to the need to introduce such assessment practices and to changes in teaching required to introduce mathematical modeling.

INTRODUCTION

Student's ability to understand the context of a situation is essential in different stages of mathematical modeling. In the modeling process, the context is the aspect of an individual's world in which the problems are stated and being able to work within a context place additional demands on the problem solver (OECD, 2018). From 2020 this agenda is incorporated into the new Latvian mathematics curriculum as a separate domain. Despite the inclusion of certain aspects of mathematical modeling in the previous curriculum, teachers currently lack the necessary experience in both instructing and evaluating mathematical modeling. This deficiency hampers their ability to provide effective support to students and adapt their teaching strategies accordingly. In this research, the authors analyzed various tasks and distinguished between routine word problems and problems that require a deep understanding of the problem context to provide the answer. The authors designed a tool for diagnosing students' contextual awareness based on written answers to support the teaching of mathematical modeling. The research questions addressed in this article are: to what extent does contextual awareness relate to general knowledge of mathematics and how able are students with various overall results to understand deeply the context of mathematical problems?

LITERATURE REVIEW

Mathematical modeling is a process of translating between real-world contexts and mathematics. It is a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insight into real-world phenomena (Bliss & Libertini, 2019). However, not every routine word problem requires the highest thinking skills. It is therefore essential to classify word problems and distinguish between different mental processes. Four types of word problems can be distinguished (1) "Bare Tasks" containing Purely Mathematical Symbols, (2) Prototype and Pseudo-Real Word Problems, (3) Realistic Word Problems, and (4) Authentic Word Problems (Csíkos & Csapó, 2011). Unlike the first two, the third and fourth type requires additional cognitive processes, a such as deep understanding of the context including the construction of an imaginary picture of the problem (Bliss & Libertini, 2019).

Understanding the problem is the first step in solving any task (Pólya, 1981). Solving mathematical problems always requires starting by understanding the meaning of a problem and looking for entry points to its solution. It means analyzing givens, constraints, relationships, and goals rather than simply jumping into a solution attempt (Billstein, Libeskind & Lott 2010). Understanding the context of the problem is also important for validating the results and interpreting the answer. (Bliss & Libertini, 2019). OECD program PISA in their assessments define context as an aspect of an individual's world in which the problems are placed (OECD, 2018). There is a certain distance between the individual and the situation in which the problem is stated and each student's ability to interact in different contexts varies (OECD, 2003). There are several red flags or blockages in each of the modeling steps (Stillman et. al., 2007), and failure to understand deeply the context can trigger several of them. PISA results show that Latvian students perform lower than the OECD average starting at level 4, where the need to understand context deeply increases.

To help the teacher organize support for students learning mathematical modeling, accurate diagnosis of potential blockages is essential (Stillman et. al., 2007). Traditional assessment, when evaluating results by points, does not always provide information on specific students' abilities such as contextual awareness. Assessment of mathematical modeling should focus on the process, not the product (Bliss & Libertini, 2019). This means making observations of students' performance rather than assessing whether an answer is correct. There is a need for specific, precise criteria that inform about student performance (Muñoz & Guskey, 2015). This kind of information about a student's performance allows the teacher to plan the support needed.

METHODOLOGY

The authors draw on a larger study that was designed to develop a diagnostic assessment system in grade 7. It consists of three separate tests, two in mathematics and one in science. The total number of items is 36. They assess students' ability to use mathematics. To ensure a representative study sample schools were selected based on the school's overall performance in the previous year's (2021) national-level assessment in grade 6th in mathematics. To answer the research questions, to what extent does contextual awareness relate to general knowledge of mathematics, and how students with various overall results delve into the context of mathematical problems, a certain number of items and a set of students were selected.

The total student sample size for this research was 200. To represent the overall performance of the students (high, above average, below average, and low), four groups were developed based on total scores in all three tests using standard deviations above and below the mean and then randomly selected 50 students from each group to include in the sample.

In four items, students' ability to use the required calculations was evaluated three times, and students' context awareness was observed three times. The sets of results in these criteria are denoted by M1, M2, and M3 for doing calculations and C1, C2, and C3 for context awareness (Table 1).

Result set	Assessment criteria
M1	The student calculates the unknown term of the proportion correctly or with minor incorrectness, when there is given redundant information
M2	Students use linear function equation to calculate water temperature.
M3	The student calculates the unknown term of the proportion correctly or with minor incorrectness, when the instructions implicitly indicate a solution path
C1	Demonstrates the ability to make assumptions appropriate to the context and given situation.
C2	Demonstrates the ability to spot the limitations of a given situation.
C3	Students explain a linear process described in the situation.

Table 1: **Assessment criteria**

Pearson’s correlation coefficients (*r*) were calculated between student results in each criterion M1, M2, M3, C1, C2, and C3. It had two objectives: (1) to ascertain the relationship between coefficients within one group, and (2) to determine the relationship between criteria in both groups. The authors established the presence of the targeted ability when at least two out of the three assessment criteria yielded positive results.

Based on the results, four student performance profiles were formed. The number of students demonstrating the profiles in each of the established student groups was analyzed. In this this study the main focus was on the number of students with high or above-average overall performance and no contextual awareness and the number of students who show contextual awareness but did not do the necessary calculations.

RESULTS AND DISCUSSION

Calculated Pearson correlation coefficients (r) showed a statistically significant correlation between results in one criterion group – M1, M2, and M3. For criteria C1, C2 and C3, there is a statistically significant correlation between C1 and C3, and between C1 and C2. Such results allow us to separate these two groups of criteria and suggest that these criteria represent two distinct capabilities: the ability to do necessary calculations and the ability to deeply understand the context of the problem. Four performance profiles emerged (Table 1).

Performance profile/Total test score	High	Above average	Below average	Low
Does calculations, show contextual awareness	27	15	4	3
Does calculations, with no contextual awareness	19	9	7	0
Does not do calculations, but show contextual awareness	3	8	10	1
Does not do calculations, and show no contextual awareness	1	18	29	46

Table 2: **Number of student performance profiles in each student group.**

The results show that there were 19 (38%) students with high total test scores who did the calculations but did not demonstrate the ability to understand the context. The situation is similar for the group with an overall score above average. There were 15 (30%) students who both did the calculations and showed contextual understanding and 9 (18%) students who did the calculations but did not show contextual awareness. We assume that these results partly confirm the results of Latvian students on PISA tests, which indicate that Latvian students perform better than average on low-level tasks but below average on the 4th level and above (OECD, 2019). High-level tasks require a deep understanding of the context of the situation and involve understanding the constraints and being able to make assumptions (Shiel, et al, 2007). As one of the methods to improve the ability to delve into problem context, Wimbley describes how verbalizing problems aids in a better understanding of the problem (Whimbey, Lochhead & Narode, 2013). The number of students who did not do calculations but demonstrated an understanding of the content is highest in the group with below-average total scores. Further research is needed on the learning needs of these students to achieve better results in mathematics. It is possible that these students learn mathematical concepts better through problem-solving and mathematical modeling approaches.

CONCLUSIONS

The assessment tool developed and validated in this study clearly shows the importance of contextual awareness in mathematics. Using this tool, it is possible to diagnose that students learning to solve problems with real context may have different needs that traditional mathematics assessment tools cannot grasp. Using the tool in everyday practice would benefit students in the long term, as for students who struggle with mathematics but cope with the context.

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